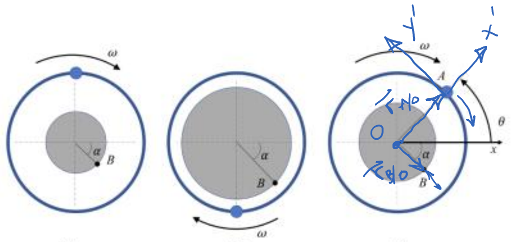
A breathing exercise video graphic (somewhat similar to this one) shows a small circle moving in a constrained circular path (constant radius 80 cm) at a **constant** angular velocity of 0.4 rad/s around an expanding and contracting inner circle. The inner circle expands and contracts sinusoidally, from a minimum radius of 30 cm to a maximum radius of 60 cm. The distance from the centre of the inner circle to a point on the edge of the inner circle can be described by the equation *r*=0.45-0.15 *sinθ* (in m), where *θ* is the position of the small circle (zero at *x*-axis).

Find the velocity and acceleration of point *B* on the edge of the inner circle as viewed by an observer on the small circle at point *A* (Fig. (c)). *θ*=45°, *α*=45°



(a) Minimum size of inner circle with location of small circle

(b) Maximum size of inner circle with location of small circle

(c)

$r = 0.45 - 0.15\sin\theta$

$\vec\omega = \vec\Omega = \dot\theta(-\hat k) = -\omega\hat k'$

$\dot{\vec\Omega} = \ddot\theta = \vec\alpha = 0$ (ω constant)

$\theta = 45°$

Fixed frame

Kin of B: $\vec r_{B/O} = r(-\hat j') = -0.45 + 0.15\sin\theta\,\hat j'$

$\vec v_B = \dot r(-\hat j') = +0.15\cos\theta\,\dot\theta\,\hat j' = 0.15\cos\theta(-\omega)\hat j'$

$= -\frac{0.15\omega}{\sqrt2}\hat j'$

$\vec a_B = \ddot r(-\hat j) = -0.15\sin\theta\,\dot\theta^2\hat j' = -0.15\sin\theta(-\omega)^2\hat j'$

$(\ddot\theta = 0)$

$= -\frac{0.15\omega^2}{\sqrt2}\hat j'$

Position: $\vec r_{A/O} = 0.8\hat i'$ m

$\vec r_{B/A} = -\vec r_{A/O} + \vec r_{B/O} = -0.8\hat i' - \overbrace{\left(0.45 - \frac{0.15}{\sqrt2}\right)}^{0.344}\hat j'$

Kin. of A: $\vec v_A = \vec\omega \times \vec r_{A/O} = -0.8\omega\hat j'$

$\vec a_A = -\omega^2\vec r_{A/O} = -0.8\omega^2\hat i'$ $(\alpha = 0)$

Rotating frame velocity

want this

$\vec v_B = \vec v_A + \vec\Omega\times\vec r_{B/A} + (\vec v_{B/A})_{rot}$

$-\frac{0.15\omega}{\sqrt2}\hat j' = -0.8\omega\hat j' + (-\omega\hat k')\times(-0.8\hat i' - 0.344\hat j') + (\vec v_{B/A})_{rot}$

$-\frac{0.15\omega}{\sqrt2}\hat j' = -\cancel{0.8\omega}\hat j' + \cancel{0.8\omega}\hat j' - 0.344\omega\hat i' + (\vec v_{B/A})_{rot}$

$(\vec v_{B/A})_{rot} = 0.344\omega\hat i' - \frac{0.15\omega}{\sqrt2}\hat j'$ $\qquad \omega = 0.4$ rad/s

$\boxed{(\vec v_{B/A})_{rot} = 0.138\hat i' - 0.042\hat j' \text{ m/s}}$

(expressed in $\hat i$ & $\hat j$
$(\vec v_{B/A})_{rot} = 0.127\hat i + 0.068\hat j$ m/s)

Rotating frame acceleration

want this

$\vec a_B = \vec a_A + \overbrace{\cancel{\dot{\vec\Omega}}}^{0}\times\vec r_{B/A} - \Omega^2\vec r_{B/A} + 2\vec\Omega\times(\vec v_{B/A})_{rot} + (\vec a_{B/A})_{rot}$

since ω = constant

$-\frac{0.15\omega^2}{\sqrt2}\hat j' = -0.8\omega^2\hat i' - (-\omega)^2(-0.8\hat i' - 0.344\hat j') + 2(-\omega\hat k')\times(0.138\hat i' - 0.042\hat j') + (\vec a_{B/A})_{rot}$

$-\frac{0.15\omega^2}{\sqrt2}\hat j' = -\cancel{0.8\omega^2}\hat i' + \cancel{0.8\omega^2}\hat i' + 0.344\omega^2\hat j' - 2\omega(0.138)\hat j' - 2\omega(0.042)\hat i' + (\vec a_{B/A})_{rot}$

$(\vec a_{B/A})_{rot} = 2\omega(0.042)\hat i' + \left(2\omega(0.138) - \frac{0.15\omega^2}{\sqrt2} - 0.344\omega^2\right)\hat j'$

$\boxed{(\vec a_{B/A})_{rot} = 0.034\hat i' + 0.038\hat j' \text{ m/s}^2}$

(expressed $\hat i$-$\hat j$
$(\vec a_{B/A})_{rot} = -0.003\hat i + 0.051\hat j$ m/s²)